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# MOMENTUM AND HEAT TRANSFER IN POWER-LAW FLUID FLOW OVER TWO-DIMENSIONAL OR AXISYMMETRICAL BODIES

H. W. KIM
Rochester Institute of Technology, Rochester, NY 14623, U.S.A.

and

D. R. JENG and K. J. DEWITT
University of Toledo, Toledo, OH 43606, U.S.A.

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**Abstract**—Momentum and heat transfer in power-law fluid flow over arbitrarily shaped two-dimensional or axisymmetrical bodies are examined theoretically. The Merk type of series expansion technique is used for the momentum analysis. For convective heat transfer, a generalized coordinate transformation is employed to analyze the temperature field in a laminar boundary layer for the body with a step change in the surface temperature distribution. In both momentum and heat transfer, the solution to the governing equations are obtained as universal functions which are independent of the geometry of the problem. The numerical and closed-form solution to the universal functions are found and then applied to analyze wedge flow and flow over a circular cylinder.

## NOMENCLATURE

| | |
|---|---|
| $a(\xi)$, | $f''(\xi, 0)$; |
| $b$, | parameter defined in equation (35); |
| $c$, | parameter defined in equation (34); |
| $C$, | constant in wedge flow; |
| $C_f$, | friction coefficient; |
| $c_p$, | specific heat at constant pressure; |
| $f$, | dimensionless stream function; |
| $H(x-x_0)$ | Heaviside function; |
| $k$, | thermal conductivity; |
| $K$, | consistency index for non-Newtonian viscosity; |
| $L$, | characteristic length; |
| $m$, | $\beta/(2-\beta)$; |
| $n$, | power-law exponent; |
| $Nu$, | Nusselt number, $q_w L/k(T_w - T_\infty)$; |
| $Nu_x$, | local Nusselt number, $q_w x/k(T_w - T_\infty)$; |
| $Pr$, | generalized Prandtl number, $\rho c_p U_\infty L/k(Re)^{2/(n+1)}$; |
| $Pr_x$, | generalized Prandtl number for wedge flow defined in equation (69); |
| $q$, | heat flux; |
| $r$, | radius of an axisymmetrical body at the point $x$; |
| $Re$, | generalized Reynolds number, $\rho U_\infty^{2-n} L^n/K$; |
| $Re_x$, | generalized Reynolds number for wedge flow, $\rho U_e^{2-n} x^n/K$; |
| $T$, | temperature; |
| $u$, | velocity component in $x$ direction; |
| $U_e$, | velocity just outside boundary layer; |
| $U_\infty$, | free stream velocity; |
| $v$, | velocity component in $y$ direction; |
| $x$, | streamwise coordinate measured along surface from forward stagnation point; |
| $x_0$, | location in $x$ direction where the wall temperature has a discontinuity; |
| $X$, | transformed dimensionless coordinate; |
| $y$, | coordinate normal to surface. |

Greek symbols

| | |
|---|---|
| $\alpha$, | thermal diffusivity; |
| $\beta$, | included wedge angle; |
| $\Gamma(n,x)$, | incomplete Gamma function, $\int_0^x e^t t^{n-1}\,dt$; |
| $\delta$, | $x/L$ in the flow over a circular cylinder; |
| $\zeta$, | transformed dimensionless coordinate; |
| $\eta$, | dimensionless coordinate; |
| $\theta$, | dimensionless temperature; |
| $\Lambda$, | wedge variable; |
| $\xi$, | dimensionless coordinate; |
| $\rho$, | density; |
| $\tau_{xy}$, | shear stress; |
| $\tau_w$, | shear stress at wall; |
| $\psi$, | stream function. |

## 1. INTRODUCTION

THE TRANSPORT phenomenon in power-law fluid flow has been investigated in several articles recently due to the frequent use of this type of fluid in modern industry. The power-law model, which the present analysis is concerned with, belongs to the group of fluids categorized as the time-independent non-Newtonian fluids. It can also encompass Newtonian fluids by virtue of its expression of the shear stress in terms of the shear rate.

Despite much effort, the extreme difficulty encountered in the boundary-layer analysis, due to the high degree of non-linearity in the momentum equation, still invites a general yet simple method for analyzing power-law fluids. The initial investigation of the external boundary layer in power-law fluid flow has been made [1]. A theoretical analysis was presented for the laminar flow past an arbitrary 2-dim. surface by using the similarity transformation. The numerical analysis of the velocity field for flow past a horizontal flat plate was given. Using a linear velocity profile, as in a similar analysis [2], the partial differential energy equation could be transformed to an ordinary differential equation. Since then, the similarity transformation for power-law fluids has been further investigated [3–8]. The similar ordinary differential equation for the velocity field in a wedge flow has been shown [9]. The authors also solved the energy equation using a generalized coordinate transformation initiated for Newtonian fluids [10].

In his paper, Merk [11] devised a new technique for analyzing the laminar boundary-layer transfer for a submerged body in Newtonian fluid flow by using the "wedge method" propounded by Meksyn [12]. Merk's transformation enabled him to reduce the boundary layer equation to the ordinary differential equation of identical form to one obtained earlier [13]. The main difference between them, however, lies in the solution technique for the equation, which was explained well in ref. [14]. The accuracy of Merk's expansion method drew the attention of some authors [15, 16]. Recently, Lin *et al.* [17] examined the velocity field for power-law fluids using the Merk-type series expansion. Only the similarity solution, i.e. the $f_0$ function for the momentum transfer was presented. No attempt has been made to obtain the remaining universal functions and heat transfer functions.

In the present analysis, the Merk series expansion method is employed for the momentum analysis. Then the temperature field and the rate of heat transfer are investigated for the same problem with a step in the surface temperature distribution by employing the generalized coordinate transformation.

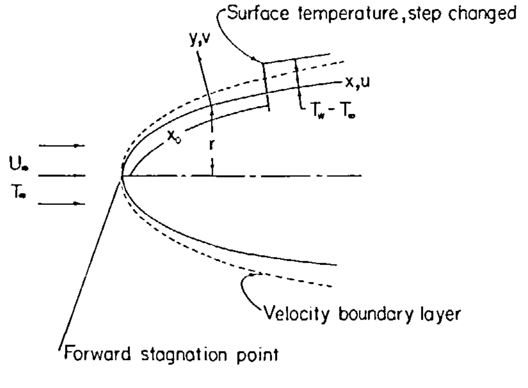


FIG. 1. Physical model and coordinate system.

## 2. FORMULATION OF GOVERNING EQUATION

The assumptions used for the boundary-layer analysis of power-law fluid flow may be stated as follows:

(a) The fluid is incompressible and all physical quantities are constant.

(b) The boundary layer is steady laminar flow, and the flow outside the boundary layer is a potential flow.

(c) There are no external body forces.

(d) The Mach number is small, and heat conduction in the $x$-direction is neglected. A physical model with the coordinate system is shown in Fig. 1.

Under the above assumptions, the boundary layer equations can be expressed in general form as follows:

*Continuity equation*

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0. \tag{1}$$

*Momentum equation*

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \frac{1}{\rho}\frac{\partial}{\partial y}(\tau_{xy}). \tag{2}$$

with the boundary conditions

$$\begin{aligned} u = v = 0 &\quad \text{at } y = 0, \\ u = U_\infty &\quad \text{at } x = 0,\ y > 0, \\ u = U_e(x) &\quad \text{at } y \to \infty \end{aligned} \tag{3}$$

where $U_e(x)$ is the velocity of the mainstream at the outer edge of the boundary layer. For the power-law model, the shear stress can be expressed as

$$\tau_{xy} = K\left(\frac{\partial u}{\partial y}\right)^n \tag{4}$$

where $n$ is called power-law exponent which is a non-negative dimensionless index parameter, and $K$ is a consistency index for non-Newtonian viscosity which is also a non-negative but dimensional quantity.

*Energy equation.*

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{5}$$

with the thermal boundary conditions

$$\begin{aligned} T(x, 0) &= T_\infty + (T_w - T_\infty)H(x - x_0), \\ T(0, y > 0) &= T_\infty, \\ T(x, \infty) &= T_\infty \end{aligned} \tag{6}$$

where $\alpha$ is the thermal diffusivity and $H(x - x_0)$ is the Heaviside function.

### 3. SOLUTION METHOD OF THE MOMENTUM EQUATIONS

In order to satisfy the continuity equation, the stream function $\psi(x, y)$ is introduced such that

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, \quad v = -\frac{1}{r}\frac{\partial\psi}{\partial x}. \tag{7}$$

The $x - y$ coordinate system is transformed into a dimensionless system by adopting new dimensionless variables

$$\xi = \frac{nK}{\rho} U_x^{n-2} L^{-n} \int_0^x \left(\frac{U_e}{U_x}\right)^{2n-1} \left(\frac{r}{L}\right)^{n+1} \left(\frac{dx}{L}\right), \tag{8}$$

$$\eta = \left[\frac{1}{(n+1)\xi}\right]^{1/(n+1)} \frac{U_e}{U_x}\frac{r}{L}\frac{y}{L}.$$

The stream function also can be nondimensionalized by defining

$$\psi = [(n+1)\xi]^{1/(n+1)} U_x L^2 f(\xi, \eta). \tag{9}$$

By using new variables, the velocity components in the boundary layer can be expressed as

$$u = U_e \frac{\partial f}{\partial \eta},$$

$$v = -\frac{nK}{\rho r}\left(\frac{U_x}{L}\right)^{n-1}\left(\frac{U_e}{U_x}\right)^{2n-1}\left(\frac{r}{L}\right)^{n+1} \times [(n+1)\xi]^{-n/(n+1)}\left\{f + (n+1)\xi\frac{\partial f}{\partial \xi} + \left[\Lambda + \frac{(n+1)\xi}{r}\frac{dr}{d\xi} - 1\right]\eta\frac{\partial f}{\partial \eta}\right\}. \tag{10}$$

The momentum equation is transformed into

$$f'''(f'')^{n-1} + ff'' + \Lambda(1 - f'^2) = (n+1)\xi\frac{\partial(f', f)}{\partial(\xi, \eta)} \tag{11}$$

where the primes stand for differentiation with respect to $\eta$ and

$$\frac{\partial(f', f)}{\partial(\xi, \eta)} = \frac{\partial f'}{\partial \xi}f' - f''\frac{\partial f}{\partial \xi}$$

is the Jacobian.

For 2-dim. flow, $r$ is simply replaced by $L$ in all the above equations. The corresponding boundary conditions become

$$f(\xi, 0) = f'(\xi, 0) = 0,$$

$$f'(\xi, \infty) = 1. \tag{12}$$

The wedge parameter, $\Lambda$, appearing in equation (11) is a function of $\xi$, i.e. $x$ only, and can be expressed as

$$\Lambda = \frac{(n+1)\xi}{U_e}\frac{dU_e}{d\xi} = \frac{(n+1)\xi}{U_e}\frac{\rho}{nK}\left(\frac{U_e}{U_x}\right)^{1-2n} U_x^{2-n} L^{n+1}\left(\frac{r}{L}\right)^{n+1}\frac{dU_e}{dx} \tag{13}$$

By employing the Merk–Meksyn procedure, $f(\xi, \eta, n)$ can be expanded in a series form as

$$f(\xi,\eta,n) = f_0(\Lambda,\eta,n) + (n+1)\xi\frac{d\Lambda}{d\xi}f_1(\Lambda,\eta,n) + (n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}f_2(\Lambda,\eta,n) + \left[(n+1)\xi\frac{d\Lambda}{d\xi}\right]^2 f_3(\Lambda,\eta,n) + \ldots \tag{14}$$

Substituting $f$ from equation (14) into equation (11) and arranging the terms which are free of $(d\Lambda/d\xi)$, and then terms with $(n+1)\xi(d\Lambda/d\xi)$, $(n+1)^2\xi^2(d^2\Lambda/d\xi^2)$, and $[(n+1)(d\Lambda/d\xi)]^2$ ..., respectively, the following set of sequential differential equations is obtained:

$$f_0''' + f_0(f_0'')^{2-n} + \Lambda(1 - f_0'^2)(f_0'')^{1-n} = 0 \tag{15}$$

$$f_1''' + (2-n)f_0(f_0'')^{1-n}f_1'' + \Lambda(1-n)(1 - f_0'^2)(f_0'')^{-n}f_1'' - (2\Lambda + n + 1)(f_0'')^{1-n}f_0'f_1' + (n+2)(f_0'')^{2-n}f_1 = (f_0'')^{1-n}\frac{\partial(f_0', f_0)}{\partial(\Lambda, \eta)}, \tag{16}$$

$$f_2''' + (2-n)f_0(f_0'')^{1-n}f_2'' + \Lambda(1-n)(1 - f_0'^2)(f_0'')^{-n}f_2'' - 2(\Lambda + n + 1)(f_0'')^{1-n}f_0'f_2' + (2n+3)(f_0'')^{2-n}f_2 = (f_0'')^{1-n}[f_0'f_1' - f_0''f_1], \tag{17}$$

$$f_3''' + (2-n)(f_0'')^{1-n}f_0f_3'' + \Lambda(1-n)(1 - f_0'^2)(f_0'')^{-n}f_3'' - 2(\Lambda + n + 1)(f_0'')^{1-n}f_0'f_3' + (2n+3)(f_0'')^{2-n}f_3$$
$$= (n+2)(n-2)(f_0'')^{1-n}f_1f_1'' + (\Lambda + n + 1)(f_0'')^{1-n}f_1'^2 + \left(\frac{\Lambda}{2} - 1\right)(n-1)(n-2)(f_1'')^2(f_0'')^{-n}f_0$$

$$+\left[\frac{(n-1)(n-2)}{2} - \Lambda(n-1)^2\right](1-f_0'^2)(f_0'')^{-(1+n)}(f_1'')^2$$

$$+(1-n)(2\Lambda+n+1)(f_0'')^{-n}f_1''f_1'f_0'$$

$$+(1-n)(f_0'')^{-n}f_1''\frac{\partial(f_0',f_0)}{\partial(\Lambda,\eta)}$$

$$+(f_0'')^{1-n}\frac{\partial(f_0',f_1)}{\partial(\Lambda,\eta)}+(f_0'')^{1-n}\frac{\partial(f_1',f_0)}{\partial(\Lambda,\eta)} \qquad (18)$$

with the corresponding boundary conditions

$$f_0(\Lambda,0,n)=f_0'(\Lambda,0,n)=0;\quad f_0'(\Lambda,\infty,n)=1,$$
$$f_1(\Lambda,0,n)=f_1'(\Lambda,0,n)=f_1'(\Lambda,\infty,n)=0,$$
$$f_2(\Lambda,0,n)=f_2'(\Lambda,0,n)=f_2'(\Lambda,\infty,n)=0,$$
$$f_3(\Lambda,0,n)=f_3'(\Lambda,0,n)=f_3'(\Lambda,\infty,n)=0.$$

With any given value of $\Lambda$, which is fixed at any streamwise location $x$, the above equations can be regarded as ordinary differential equations and the solutions to these equations become universal.

There may not be any analytic solutions to the above equations, and thus as the solution method one must resort to numerical analysis. The classical fourth-order Runge–Kutta method was employed with integrating step-size control. Since the method commonly used in the initial-value problem was adopted in this asymptotic type of boundary-value problem, the assumed initial-value of $f_i''(0)$ should be provided and the resulting solution must satisfy the remaining boundary conditions, i.e. its first derivative and an additional condition which is the second derivative at the outer edge of the boundary layer. The additional boundary condition $f_i''(\infty)$ is required due to the nature of the asymptotes in the solution. In the practical sense, the initial value of $f_i''(0)$ cannot be known exactly so that an iteration procedure must be used. The Newton–Raphson technique was used for this purpose. A difficulty encountered in this numerical integration method is a singularity at infinity in equation (15) due to the third term of the equation for dilatant fluids when $n$ is between 1.0 and 2.0. This type of singularity, however, can be removed if the limiting process is adopted. The L'Hospital Rule was applied for this process with control of the integrating step size. When $n$ is greater than 2, equation (15) represents the two point boundary-value problem which means the $\eta_\infty$ being finite, as pointed out by Acrivos *et al.* [1]. This phenomenon is attributed to the power-law exponent, $n$, and hence is not encountered in either Newtonian fluids or pseudoplastic fluids.

To obtain numerical results, several physical models of fluids are considered. 23.3% Illinois yellow clay in water ($n = 0.229$), 0.67% CMC in water ($n = 0.520$), and 10% napalm in kerosene ($n = 0.716$) for pseudoplastic fluids. Newtonian fluid ($n = 1.0$) and ethylene oxide in sodium chloride solution ($n = 1.2, 1.6, 2.0$) for dilatant fluids are also used. The choice of the upper limit of $n$ was based on the fact that most power-law fluids have the value of $n$ less than two.

A general computer program was developed for the first three equations. Although the velocity profile can be obtained from the computer program, the main interest is in the velocity gradients at the wall, in order to obtain the drag on the body and the rate of heat transfer. For this purpose, the numerical results of $f_0''(0)$, $f_1''(0)$, and $f_2''(0)$ for the given values of $n$ and $\Lambda$ are tabulated in Table 1. They are compared with those of Chao [14] when $n$ is unity. It is found that the values of $f_0''(0)$ in both analyses agree to 11 digits and those of $f_1''(0)$ and $f_2''(0)$ differ by 0.25% at most. The numerical results of $f_0''(0)$ for non-Newtonian fluids are compared with those of refs. [8, 9]. The maximum discrepancy between them is 4% and 0.2%, respectively.

Once the velocity functions are known, the local friction coefficient can be readily calculated. Since the shear stress was defined in equation (4), the shear stress at the wall is given by

$$\tau_w = K\left(\frac{\partial u}{\partial y}\right)^n\bigg|_{y=0}$$
$$= K\left[\frac{1}{(n+1)\xi}\right]^{n/(n+1)}\left(\frac{U_e^2}{U_\infty L}\right)^n\left(\frac{r}{L}\right)^n$$
$$\times\left[f_0''(0)+(n+1)\xi\frac{d\Lambda}{d\xi}f_1''(0)+(n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}f_2''(0)+\ldots\right]^n. \qquad (19)$$

Defining the local friction coefficient by $C_f = \tau_w/(\rho U_\infty^2/2)$ then

$$C_f = 2\left[\frac{1}{(n+1)\xi}\right]^{1/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2n}\frac{1}{Re}\left(\frac{r}{L}\right)^n\left[f_0''(0)+(n+1)\xi\frac{d\Lambda}{d\xi}f_1''(0)+(n+1)^2\xi^2\frac{d^2\Lambda}{d\xi^2}f_2''(0)+\ldots\right]^n \qquad (20)$$

where

$$Re = \rho U_\infty^{2-n}L^n/K$$

is a generalized Reynolds number.

## 4. SOLUTION OF THE ENERGY EQUATION

In order to solve the heat transfer problem with a step change in the surface temperature distribution, further coordinate transformation for the energy equation is needed. For this purpose the new dimensionless

Table 1. Numerical results of $f_i''(0)$ for power-law fluids

| $n$ | $\Lambda$ | $f_0''(0)$ | $f_1''(0) \times 10$ | $f_2''(0) \times 10^2$ |
|---|---|---|---|---|
| | 2.68341 | 1.9568071 | − 0.1502860 | 0.0886065 |
| | 2.0 | 1.5565997 | − 0.2028859 | 0.1474157 |
| | 1.5 | 1.2485316 | − 0.2685225 | 0.2350089 |
| | 1.0 | 0.9221991 | − 0.3874294 | 0.4259752 |
| | 0.75 | 0.7494606 | − 0.4903864 | 0.6180423 |
| 0.229 | 0.50 | 0.5676960 | − 0.6565673 | 0.9682217 |
| | 0.25 | 0.3731422 | − 0.9600644 | 1.7011382 |
| | 0.00 | 0.1585939 | − 1.6278791 | 3.5851544 |
| | − 0.10 | 0.0646986 | − 2.1474721 | 5.2446473 |
| | − 0.15 | 0.0160405 | − 2.6003656 | 6.7798058 |
| | 1.46154 | 1.36246393 | − 0.21494049 | 0.17050786 |
| | 1.25 | 1.23990722 | − 0.25247219 | 0.21780692 |
| | 1.00 | 1.08656532 | − 0.31420028 | 0.30224534 |
| | 0.75 | 0.92140437 | − 0.40784471 | 0.44311522 |
| 0.520 | 0.50 | 0.73995249 | − 0.56329145 | 0.70236620 |
| | 0.25 | 0.53374449 | − 0.86088559 | 1.25704889 |
| | 0.00 | 0.28193462 | − 1.61881113 | 2.84021463 |
| | − 0.10 | 0.15386494 | − 2.43663056 | 4.63630326 |
| | − 0.15 | 0.07461074 | − 3.45034669 | 6.81761648 |
| | 1.19832 | 1.27454773 | − 0.22412658 | 0.18268289 |
| | 1.00 | 1.16036533 | − 0.26933531 | 0.23835769 |
| | 0.75 | 1.00414043 | − 0.35337571 | 0.35052194 |
| | 0.50 | 0.82904080 | − 0.49456703 | 0.55770961 |
| 0.716 | 0.25 | 0.62451676 | − 0.77061857 | 1.00621228 |
| | 0.00 | 0.36313757 | − 1.51689628 | 2.35484749 |
| | − 0.10 | 0.22193012 | − 2.41167773 | 4.06219319 |
| | − 0.15 | 0.12961836 | − 3.62337789 | 6.37719317 |
| | 1.0 | 1.232587657 | − 0.21495486 | 0.1701345 |
| | 0.75 | 1.090441562 | − 0.28526627 | 0.2509169 |
| | 0.50 | 0.927680040 | − 0.40504497 | 0.4009941 |
| 1.0 | 0.25 | 0.731940849 | − 0.64497584 | 0.7311176 |
| | 0.0 | 0.469599988 | − 1.33284826 | 1.7790514 |
| | − 0.1 | 0.319269760 | − 2.22364220 | 3.2220740 |
| | − 0.15 | 0.216361406 | − 3.47115850 | 5.2949057 |
| | 1.0 | 1.26641921 | − 0.18467854 | 0.13594487 |
| | 0.91667 | 1.22409277 | − 0.20213430 | 0.15370956 |
| | 0.75 | 1.13386852 | − 0.24654164 | 0.20076631 |
| 1.2 | 0.50 | 0.98034168 | − 0.35276925 | 0.32166537 |
| | 0.25 | 0.79280349 | − 0.56842709 | 0.59013672 |
| | 0.0 | 0.53506307 | − 1.20450065 | 1.46453568 |
| | − 0.10 | 0.38295155 | − 2.05212958 | 2.70380187 |
| | − 0.15 | 0.27664817 | − 3.23918625 | 4.48784407 |
| | 1.0 | 1.3074729 | − 0.1389691 | 0.0896980 |
| | 0.8125 | 1.2225340 | − 0.1728273 | 0.1196185 |
| | 0.75 | 1.1920803 | − 0.1872365 | 0.1328106 |
| 1.6 | 0.50 | 1.0560872 | − 0.2712033 | 0.2138341 |
| | 0.25 | 0.8860135 | − 0.4453047 | 0.3966827 |
| | 0.0 | 0.6433860 | − 0.9798961 | 1.0146403 |
| | − 0.10 | 0.4940646 | − 1.7179589 | 1.9224022 |
| | − 0.15 | 0.3868320 | − 2.7473601 | 3.2270801 |
| | 1.0 | 1.326903 | − 0.107393 | 0.061742 |
| | 0.75 | 1.225727 | − 0.145714 | 0.091659 |
| | 0.60 | 1.156237 | − 0.180923 | 0.120725 |
| 2.0 | 0.50 | 1.104986 | − 0.213007 | 0.148203 |
| | 0.25 | 0.951416 | − 0.354769 | 0.277481 |
| | 0.0 | 0.726468 | − 0.801947 | 0.725504 |
| | − 0.10 | 0.584034 | − 1.432707 | 1.398180 |
| | − 0.15 | 0.479965 | − 2.308608 | 2.362014 |

variables, $X$, $\zeta$, and the dimensionless temperature $\theta$ are defined as

$$X = \left[1 - \left(\frac{\xi_0}{\xi}\right)^c\right]^{1/3}$$

$$\zeta = b(\xi)\frac{\eta}{X}, \tag{21}$$

$$\theta(X,\zeta) = \frac{T - T_\infty}{T_w - T_\infty}$$

where $\xi_0$ is defined by

$$\xi_0 = \frac{nk}{\rho} U_\infty^{n-2} L^{-n} \int_0^{X_0} \left(\frac{U_e}{U_\infty}\right)^{2n-1} \left(\frac{r}{L}\right)^{n+1} \frac{\mathrm{d}x}{L}. \tag{22}$$

The coordinate transformation of equation (21) was first proposed by Chao and Cheema [10] for convective heat transfer with a non-isothermal surface for wedge flow and later generalized by Jeng *et al.* [16].

On substituting $u$ and $v$ expressed by equation (10) into equation (5), the transformed energy equation becomes

$$\frac{\partial^2\theta}{\partial\zeta^2} + \left\{\frac{[(n+1)\xi]^{2/(n+1)}}{(n+1)\xi\alpha b}\left[f + (n+1)\xi\frac{\partial f}{\partial \xi}\right] + \frac{c[(n+1)\xi]^{2/(n+1)}}{3\alpha b^2\xi}\frac{(1-X^3)}{X} - \frac{[(n+1)\xi]^{2/(n+1)}}{\alpha b^3}\frac{\mathrm{d}b}{\mathrm{d}\xi}X^2\frac{\partial f}{\partial\eta}\zeta\right\}$$
$$\times \left(\frac{U_e}{U_\infty}\right)^{2n-2}\frac{nK}{\rho}\left(\frac{U_\infty r}{L^2}\right)^{n-1}\frac{\partial\theta}{\partial\zeta} - \frac{c[(n+1)\xi]^{2/(n+1)}}{3\alpha b^2\xi}$$
$$\times \left(\frac{U_e}{U_\infty}\right)^{2n-2}\frac{nk}{\rho}\left(\frac{U_\infty r}{L^2}\right)^{n-1}\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial X} = 0 \tag{23}$$

with the equivalent boundary conditions in dimensionless form

$$\theta(X,0) = 1,$$
$$\theta(X,\infty) = 0. \tag{24}$$

The dimensionless stream function $f$ is now expanded in a power series as

$$f(\xi,\eta) = \sum_{m=2}^{\infty} a_m(\xi)\frac{\eta^m}{m!}. \tag{25}$$

The coefficient appearing in the above equation can be determined by substituting $f$ from equation (25) into equation (11) and rearranging in terms of like powers of $\eta$

$$a_2(\xi) = \left.\frac{\partial^2 f}{\partial\eta^2}\right|_{\eta=0} = a,$$

$$a_3(\xi) = -\Lambda a^{1-n},$$

$$a_4(\xi) = (1-n)\Lambda^2 a^{1-2n},$$

$$a_5(\xi) = (n+1)\xi a' a^{2-n} - (1-n)(1-2n)\Lambda^3 a^{1-3n} + (2\Lambda - 1)a^{3-n},$$

$$a_6(\xi) = (1-n)(2n-1)(3n-1)\Lambda^4 a^{1-4n} + [4\Lambda(n-1)(2\Lambda-1) - 6\Lambda(6\Lambda-4) - 2(n+1)\xi\Lambda']a^{3-2n} + 6(n-1)(n+1)\xi\Lambda a' a^{2-2n} \tag{26}$$

where the primes stand for differentiation with respect to $\xi$. The unknown function $a$ is determined by using the value of $f_1''(0)$ obtained in Section 3. Now, using the new variables $\zeta$ and $X$, $f$ can be written as

$$f = \frac{a_2(\xi)\zeta^2}{2!b^2}X^2 + \frac{a_3(\xi)\zeta^3}{3!b^3}X^3 + \frac{a_4(\xi)\zeta^4}{4!b^4}X^4 + \frac{a_5(\xi)\zeta^5}{5!b^5}X^5 + \frac{a_6(\xi)\zeta^6}{6!b^6}X^6 + \ldots \tag{27}$$

The solution to equation (23) is then expanded in series form

$$\theta(\xi,\zeta,X) = \sum_{k=0}^{\infty} \theta_k(\xi,\zeta)X^k \tag{28}$$

with

$$\theta_0(\xi,0) = 1;\ \theta_1(\xi,0) = \theta_2(\xi,0) = \theta_3(\xi,0) = \ldots = 0,$$
$$\theta_0(\xi,\infty) = \theta_1(\xi,\infty) = \theta_2(\xi,\infty) = \theta_3(\xi,\infty) = \ldots = 0. \tag{29}$$

Thus the boundary conditions in equation (24) are satisfied. By substituting $f$, $(\partial f/\partial\eta)$, $(\partial f/\partial\xi)$, and $\theta$ from equations (27) and (28) into equation (23) and equating the coefficients of like powers of $X$ with a proper choice of $c$ and $b(\xi)$, the following sequence of second-order, linear differential equations can be obtained:

$$\frac{\partial^2\theta_0}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_0}{\partial\zeta} = 0, \tag{30}$$

$$\frac{\partial^2\theta_1}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_1}{\partial\zeta} - 3\zeta\theta_1 = -\frac{3}{2}\frac{a_3}{a_2 b}\zeta^3\frac{\partial\theta_0}{\partial\zeta}, \tag{31}$$

$$\frac{\partial^2\theta_2}{\partial\zeta^2} + 3\zeta^2\frac{\partial\theta_2}{\partial\zeta} - 6\zeta\theta_2 = -\frac{a_4}{2a_2 b^2}\zeta^4\frac{\partial\theta_0}{\partial\zeta} - \frac{3a_3}{2a_2 b}\zeta^3\frac{\partial\theta_1}{\partial\zeta} + \frac{3a_3}{2a_2 b}\zeta^2\theta_1 \tag{32}$$

$$\frac{\partial^2\theta_3}{\partial\zeta^2}+3\zeta^2\frac{\partial\theta_3}{\partial\zeta}-9\zeta\theta_3=\Bigg[-\frac{a_5}{8a_2b^3}\zeta^5-\frac{9}{2(n+1)c}$$

$$\times\zeta^2+\frac{9\xi}{cb}\frac{db}{d\xi}\zeta^2-\frac{9\xi a_2'}{2ca_2}\zeta^2+\quad 3\zeta^2\Bigg]$$

$$\times\frac{\partial\theta_0}{\partial\zeta}-\frac{a_4}{2a_2b^2}\zeta^4\frac{\partial\theta_0}{\partial\zeta}$$

$$-\frac{3a_3}{2a_2b}\zeta^3\frac{\partial\theta_2}{\partial\zeta}+\frac{a_4}{2a_2b^2}\zeta^3\theta_1$$

$$+\frac{3a_3}{a_2b}\zeta^2\theta_2 \tag{33}$$

where

$$c=\frac{3}{2(n+1)} \tag{34}$$

and

$$b(\xi)=\left\{\frac{1}{3!(n+1)}[n(n+1)]^{2(n+1)}\left(\frac{Re\xi}{n}\right)^{(1-n)(n+1)}\right.$$

$$\left.\times\left(\frac{U_e}{U_\infty}\right)^{2(n-1)}\left(\frac{r}{L}\right)^{n-1}a_2Pr\right\}. \tag{35}$$

The solution to equation (30) is

$$\theta_0=1-\frac{\Gamma(1/3,\zeta^3)}{\Gamma(1/3)} \tag{36}$$

thus,

$$\theta_0'(0)=1.1198. \tag{37}$$

In order to obtain each solution to the above equations in terms of universal functions, $\theta_n$'s are rewritten as

$$\theta_1=M\bar\theta_1, \tag{38}$$

$$\theta_2=M^2\bar\theta_{2,1}+N\bar\theta_{2,2}, \tag{39}$$

$$\theta_3=M^3\bar\theta_{3,1}+MN\bar\theta_{3,2}+P\bar\theta_{3,3}+Q\bar\theta_{3,4}, \tag{40}$$

where

$$M=-\frac{3a_3}{2a_2b},$$

$$N=-\frac{a_4}{2a_2b^2},$$

$$P=-\frac{a_5}{8a_2b^3},$$

$$Q=3\left[1-\frac{3}{2(n+1)c}+\frac{3\xi}{cb}\frac{db}{d\xi}-\frac{3\xi a_2'}{2ca_2}\right]. \tag{41}$$

Therefore, the equations (31)–(33) become

$$\bar\theta_1''+3\zeta^2\bar\theta_1'-3\zeta\bar\theta_1=\zeta^3\theta_0', \tag{42}$$

$$\bar\theta_{2,1}''+3\zeta^2\bar\theta_{2,1}'-6\zeta\bar\theta_{2,1}=\frac{\zeta^7}{5}\theta_0', \tag{43}$$

$$\bar\theta_{2,2}''+3\zeta^2\bar\theta_{2,2}'-6\zeta\bar\theta_{2,2}=\zeta^4\theta_0', \tag{44}$$

$$\bar\theta_{3,1}''+3\zeta^2\bar\theta_{3,1}'-9\zeta\bar\theta_{3,1}=\zeta^3\bar\theta_{2,1}-2\zeta^2\bar\theta_{2,1}, \tag{45}$$

$$\bar\theta_{3,2}''+3\zeta^2\bar\theta_{3,2}'-9\zeta\bar\theta_{3,2}$$

$$=\zeta^4\bar\theta_1'-\zeta^3\bar\theta_1+\zeta^3\bar\theta_{2,2}-2\zeta^2\bar\theta_{2,2} \tag{46}$$

$$\bar\theta_{3,3}''+3\zeta^2\bar\theta_{3,3}'-9\zeta\bar\theta_{3,3}=\zeta^5\theta_0', \tag{47}$$

$$\bar\theta_{3,4}''+3\zeta^2\bar\theta_{3,4}'-9\zeta\bar\theta_{3,4}=\zeta^2\theta_0', \tag{48}$$

with the boundary conditions

$$\begin{aligned}\bar\theta_1(0)&=\bar\theta_1(\infty)=\bar\theta_{2,1}(0)=\bar\theta_{2,1}(\infty)\\&=\bar\theta_{2,2}(0)=\bar\theta_{2,2}(\infty)\\&=\bar\theta_{3,1}(0)=\bar\theta_{3,1}(\infty)\\&=\bar\theta_{3,2}(0)=\bar\theta_{3,2}(\infty)=\bar\theta_{3,3}(0)\\&=\bar\theta_{3,3}(\infty)=\bar\theta_{3,4}(0)=\bar\theta_{3,4}(\infty)=0.\end{aligned} \tag{49}$$

The solutions to equations (42), (47) and (48) are of the closed form

$$\bar\theta_1=\frac{\zeta}{5\Gamma(1/3)}\left[\Gamma\left(\frac{4}{3}\right)-\Gamma\left(\frac{4}{3},\zeta^3\right)\right] \tag{50}$$

then

$$\bar\theta_1'(0)=\frac{1}{15}, \tag{51}$$

$$\bar\theta_{3,3}=\frac{1}{9\Gamma(1/3)}\left(\frac{2}{3}\zeta+\zeta^4\right)e^{-\zeta^3}, \tag{52}$$

$$\bar\theta_{3,4}=\frac{1}{6\Gamma(1/3)}\zeta e^{-\zeta^3} \tag{53}$$

and

$$\bar\theta_{3,3}'(0)=\frac{2}{27\Gamma(1/3)}, \tag{54}$$

$$\bar\theta_{3,4}'(0)=\frac{1}{6\Gamma(1/3)}. \tag{55}$$

There seems to be no closed form solution to the equations (43)–(46). Hence the equations have been integrated numerically. Since equations (43) and (44) are of identical form to equations (3.39) and (3.40) in ref. [15], respectively, the numerical tabulation given there can be used for comparison. The numerical results of $\bar{\theta}_{3,1}$ and $\bar{\theta}_{3,2}$ are tabulated in Table 2. The first derivatives of the above numerical solutions at $\zeta = 0$ are

$$\bar{\theta}'_{2,1}(0) = 0.81748 \times 10^{-2}, \tag{56}$$

$$\bar{\theta}'_{2,2}(0) = 0.40872 \times 10^{-1}, \tag{57}$$

$$\bar{\theta}'_{3,1}(0) = 0.17205 \times 10^{-2}, \tag{58}$$

$$\bar{\theta}'_{3,2}(0) = 0.12904 \times 10^{-1}. \tag{59}$$

then

$$q_w = k(T_w - T_\infty)\left[\frac{1}{(n+1)\xi}\right]^{1/(n+1)} \frac{U_e}{U_\infty L} \times \left[\frac{(n^2+n)^{2/(n+1)}}{6(n+1)}\left(\frac{Re\,\xi}{n}\right)^{(1-n)/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2(n-1)} \times \left(\frac{r}{L}\right)^{n-1} Pr\, a\right]^{1/3} X^{-1}\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} \tag{61}$$

where

$$\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} = 1.1198 - \frac{1}{10}\frac{\Lambda}{a^n}\left[\frac{a}{6(n+1)} \times (n^2+n)^{2/(n+1)}\left(\frac{Re\,\xi}{n}\right)^{(1-n)/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2(n-1)} \times \left(\frac{r}{L}\right)^{(n-1)} Pr\right]^{-1/3} X$$
$$- \left[\frac{9}{4}(0.0081748) - \frac{(1-n)}{2} \times (0.040874)\right]\left(\frac{\Lambda}{a^n}\right)^2$$
$$\left[\frac{a}{6(n+1)}(n^2+n)^{2/(n+1)} \times \left(\frac{Re\xi}{n}\right)^{(1-n)/(1+n)}\left(\frac{U_e}{U_\infty}\right)^{2n-2}\left(\frac{r}{L}\right)^{n-1} Pr\right]^{-2/3} X^2$$
$$- \left\{0.062214\left[2(n-1)(2\Lambda - 1) - (n+1)\xi\frac{a'}{a} + 2(n^2-1)\frac{\xi}{r}\frac{dr}{d\xi}\right]\right.$$
$$+ \left[\frac{27}{8}(0.0017205) - \frac{3(1-n)}{4} \times (0.012904)\right]\left(\frac{\Lambda}{a^n}\right)^3 \frac{6(n+1)}{a}(n^2+n)^{-2/(n+1)}$$
$$\left(\frac{Re\xi}{n}\right)^{(1-n)/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2(1-n)}\left(\frac{r}{L}\right)^{1-n} Pr^{-1} - \frac{1}{8}(0.02765)$$
$$\left[(2\Lambda - 1)a^{2-n} - (1-n)(1-2n)\left(\frac{\Lambda}{a^n}\right)^3 + (n+1)\xi a^{1-n} a'\right]$$
$$\frac{6(n+1)}{a}(n^2+n)^{-2/(n+1)}\left(\frac{Re\xi}{n}\right)^{(n-1)/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2(1-n)}$$
$$\left.\left(\frac{r}{L}\right)^{1-n} Pr^{-1}\right\} X^3 + \dots \tag{62}$$

The dimensionless temperature in boundary layer is, therefore, written as

$$\theta(\xi,\zeta,X) = \theta_0 + M\bar{\theta}_1 X + (M^2\bar{\theta}_{2,1} + N\bar{\theta}_{2,2})X^2 + (M^3\bar{\theta}_{3,1} + MN\bar{\theta}_{3,2} + P\bar{\theta}_{3,3} + Q\bar{\theta}_{3,4})\cdot X^3 + \dots \tag{60}$$

The local heat flux at the wall is given by

$$q_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0},$$

Equation (62) is identical to that of Jeng *et al.* [16] when $n$ is unity.

If the Nusselt number is defined by $Nu = q_w L/k(T_w - T_\infty)$, then

$$Nu\,Re^{-1/(n+1)} = \left[\frac{1}{n(n+1)}\right]^{1/(n+1)}\left(\frac{n}{Re\xi}\right)^{1/(n+1)}\left(\frac{U_e}{U_\infty}\right) \times \left[\frac{(n^2+n)}{6(n+1)}\right]^{2/(n+1)}\left(\frac{Re\xi}{n}\right)^{(1-n)/(n+1)} \times \left(\frac{U_e}{U_\infty}\right)^{2(n-1)}\left(\frac{r}{L}\right)^{n-1} Pr\, a\Big]^{1/3} X^{-1} \times \left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0}. \tag{63}$$

Table 2. Numerical results of $\bar{\theta}_{3,1}$ and $\bar{\theta}_{3,2}$ functions

| $\zeta$ | $\bar{\theta}_{3,1}(\zeta)$ | | $\bar{\theta}'_{3,1}(\zeta)$ | | $\bar{\theta}_{3,2}(\zeta)$ | | $\bar{\theta}'_{3,2}(\zeta)$ | |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 00 | 0.172047 | − 02 | 0.0 | 00 | 0.129035 | − 01 |
| 0.2 | 0.345340 | − 03 | 0.174472 | − 02 | 0.259037 | − 02 | 0.130935 | − 01 |
| 0.4 | 0.706001 | − 03 | 0.188811 | − 02 | 0.530487 | − 02 | 0.142778 | − 01 |
| 0.6 | 0.111202 | − 02 | 0.220069 | − 02 | 0.839461 | − 02 | 0.167885 | − 01 |
| 0.8 | 0.159489 | − 02 | 0.263162 | − 02 | 0.119814 | − 01 | 0.185204 | − 01 |
| 1.0 | 0.214531 | − 02 | 0.273829 | − 02 | 0.152926 | − 01 | 0.125951 | − 01 |
| 1.2 | 0.258697 | − 02 | 0.130933 | − 02 | 0.161311 | − 01 | − 0.598489 | − 02 |
| 1.4 | 0.252623 | − 02 | − 0.212250 | − 02 | 0.127948 | − 01 | − 0.257521 | − 01 |
| 1.6 | 0.178532 | − 02 | − 0.483260 | − 02 | 0.704435 | − 02 | − 0.282275 | − 01 |
| 1.8 | 0.833569 | − 03 | − 0.413904 | − 02 | 0.242826 | − 02 | − 0.158842 | − 01 |
| 2.0 | 0.240189 | − 03 | − 0.180663 | − 02 | 0.561590 | − 03 | − 0.494263 | − 02 |
| 2.2 | 0.404829 | − 04 | − 0.417781 | − 02 | 0.736835 | − 04 | − 0.850734 | − 03 |
| 2.4 | 0.383711 | − 05 | − 0.507224 | − 04 | 0.546178 | − 05 | − 0.792545 | − 04 |
| 2.6 | 0.237521 | − 06 | − 0.310993 | − 05 | 0.218863 | − 06 | − 0.387334 | − 05 |

t is noted that the term $Re\xi/n$ appearing in the above quation is equivalent to the expression

$$\int_0^x \left(\frac{U_e}{U_\infty}\right)^{2(n-1)} \left(\frac{r}{L}\right)^{n+1} \frac{dx}{L}$$

vhich is dimensionless and independent of any physi- al units.

## 5. APPLICATION OF ANALYSIS

The general expression of momentum and heat ransfer have been derived for arbitrarily shaped 2-dim. ɪr axisymmetric bodies with step-changed non- sothermal surfaces in power-law fluid flow. The .pplicability of this analysis is examined by analyzing ertain cases of geometry with a step-change in the urface temperature (including the isothermal surface) ɪy regarding it as an extremum case of the non- sothermal boundary condition. The flow geometries selected are wedge flow and flow over a circular cylinder. Examples for axisymmetrical bodies will not be presented but the analysis would be straightforward using the procedure given in this analysis.

### 5.1. *Wedge flow*

For the case of a wedge flow, the velocity at the outer edge of boundary is given by $U_e = Cx^m$ provided $m = \beta/(2-\beta)$. In the above expression, $C$ is a constant and $\beta$ is an included wedge angle divided by $\pi$. According to the transformation in Section 3,

$$\xi = \frac{C^{2n-1}}{m(2n-1)+1} \frac{nK}{\rho} (U_\infty L)^{-(n+1)} x^{m(2n-1)+1}, \tag{64}$$

$$\Lambda = \frac{m(n+1)}{m(2n-1)+1} \tag{65}$$

Therefore $\Lambda$ becomes constant. This phenomenon makes the set of equations in the series reduce to a

Table 3. $\frac{1}{2}C_f(Re_x)^{1/(n+1)}$ for wedge flow

| $\Lambda$ | $n = 0.229$ | $\Lambda$ | $n = 0.520$ | $\Lambda$ | $n = 0.716$ |
|---|---|---|---|---|---|
| 2.68341 | 1.276936 | 1.46154 | 1.290100 | 1.19832 | 1.268198 |
| 1.50 | 1.212229 | 1.25 | 1.225968 | — | — |
| 1.00 | 1.161385 | 1.00 | 1.141980 | 1.00 | 1.152096 |
| 0.75 | 1.124061 | 0.75 | 1.045738 | 0.75 | 1.004359 |
| 0.50 | 1.071923 | 0.50 | 0.930896 | 0.50 | 0.848719 |
| 0.25 | 0.991041 | 0.25 | 0.783684 | 0.25 | 0.673092 |
| 0.00 | 0.830733 | 0.00 | 0.561079 | 0.00 | 0.444320 |
| − 0.10 | 0.682246 | − 0.10 | 0.409875 | − 0.10 | 0.309080 |
| − 0.15 | 0.497885 | − 0.15 | 0.281444 | − 0.15 | 0.209237 |
| − 0.164 | 0.302594 | − 0.18 | 0.079187 | − 0.188 | 0.045900 |
| $\Lambda$ | $n = 1.2$ | $\Lambda$ | $n = 1.6$ | $\Lambda$ | $n = 2.0$ |
| 0.91667 | 1.210039 | 0.8125 | 1.173542 | 0.75 | 1.14655 |
| 0.75 | 0.975388 | 0.75 | 1.023729 | 0.60 | 0.74580 |
| 0.50 | 0.708620 | 0.50 | 0.636713 | 0.50 | 0.58699 |
| 0.25 | 0.489868 | 0.25 | 0.396683 | 0.25 | 0.33210 |
| 0.00 | 0.278048 | 0.00 | 0.205388 | 0.00 | 0.15983 |
| − 0.10 | 0.179967 | − 0.10 | 0.128046 | − 0.10 | 0.09694 |
| − 0.15 | 0.119881 | − 0.15 | 0.084551 | − 0.15 | 0.06356 |
| − 0.20 | 0.026202 | − 0.20 | 0.025249 | − 0.20 | 0.02402 |

Table 4. Numerical results of $C_f\ (Re_x)^{1/(n+1)}$ for flow past a flat plate

| $n$ | 0.229 | 0.520 | 0.716 | 1.0 | 1.2 | 1.6 | 2.0 |
|---|---|---|---|---|---|---|---|
| Acrivos *et al* [1] | 0.840 | 0.562 | 0.445 | 0.332 | 0.275 | 0.205 | 0.161 |
| Present analysis | 0.832 | 0.561 | 0.444 | 0.332 | 0.278 | 0.205 | 0.160 |

single equation, namely the $f_0$ equation which represents the local similarity. It is interesting to note that for certain geometries the values of $\Lambda$ become constant for all values of $n$. Some values of $\Lambda$ are for example, $\Lambda = 0$ for a flat plate and $\Lambda = 1/2$ for a right angle wedge. The laminar boundary layer for these cases have been examined, as mentioned in the Introduction.

Following the definition of equation (20), the local friction coefficient can be written as

$$C_f\left[\frac{\rho x^n U_\infty^{2-n}}{K}\right]^{1/(n+1)}\left(\frac{U_\infty}{U_e}\right)^{3n/(n+1)} = 2\left[\frac{m(2n-1)+1}{n(n+1)}\right]^{1/(n+1)}[f_0''(0)]^n. \quad (66)$$

If a flat plate with a length of $L$ in Newtonian flow is considered, equation (66) becomes

$$C_f\, Re^{1/2} = \frac{2}{\sqrt{2}} f''(0).$$

For both sides of the plate, $C_f\ Re^{1/2} = 1.3282$ which agrees with the solution obtained by the Blasius series.

For wedge flow especially, another expression for defining the local friction coefficient is frequently used. This is given by $C_f = \tau_w/\rho U_e^2/2$. Then,

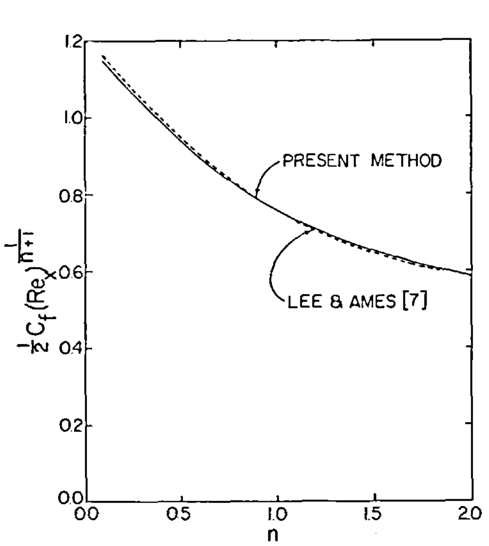


FIG. 2. Comparison of $\frac{1}{2}\ C_f\ (Re_x)^{1/(n+1)}$ for flow over a right angle wedge.

$$\tfrac{1}{2}C_f\,(Re_x)^{1/(n+1)} = \{[(n+1) - \Lambda(2n-1)]n\}^{-n/(n+1)}[f''(0)]^n \quad (67)$$

where

$$Re_x = \frac{\rho}{K}x^n U_e^{2-n} = \frac{\rho}{K}C^{2-n}x^{m(2-n)+n}. \quad (68)$$

The numerical results of $1/2\ C_f\ (Re_x)^{1/(n+1)}$ are presented in Table 3 for selected $n$'s and $\Lambda$'s. The numerical results for a right-angle wedge flow are compared with those of Lee *et al.* in Fig. 2. The results agree within 0.5%. For further examination of the accuracy, the present results for flow past a flat plate are compared with those of Acrivos *et al.* [1] and are tabulated in Table 4. It is noted that the values from Acrivos *et al.* were read from Fig. 4 in their paper. Thus they may not present the exact values which the authors originally presented. Generally, the agreement between these two analyses can be regarded as fairly good.

In addition to the drag coefficient, another important flow characteristic is the separation of flow. The separation point of wedge flow for various values of $n$ can be found by determining the value of $\Lambda$ from equation (15). The $\Lambda$ is denoted by $\Lambda_s$ which makes the velocity and its derivatives at wall identically vanish. The $\Lambda_s$ are $-0.166$, $-0.182$, $-0.190$, $-0.199$, $-0.204$, $-0.212$ and $-0.218$ for the corresponding $n$ being equal to 0.229, 0.520, 0.716, 1.0, 1.2, 1.6 and 2.0, respectively. When $n$ is unity, $\Lambda$ is equal to $-0.199$ which agrees with the separation point for Newtonian fluid obtained by Blasius series expansion. However, prediction of the separation point for power-law fluid flow over an arbitrary body by the present method may not be possible.

For the rate of heat transfer at the wall in wedge flow, the Nusselt number defined in equation (62) will be slightly modified for the direct comparison of the present results with those reported in literature. It is not difficult to show that the term

$$\left(\frac{Re\xi}{n}\right)^{(n-1)/(n+1)}\left(\frac{U_e}{U_\infty}\right)^{2(n-1)}\left(\frac{r}{L}\right)^{n-1} a\, Pr$$

in the general expression becomes

$$\left[\frac{1}{m(2n-1)+1}\right]^{(1-n)/(n+1)} a\frac{\rho c_p}{k}\left(\frac{k}{\rho}\right)^{2/(n+1)}$$

$$(C^3 x^{3m-1})^{(n-1)/(n+1)}$$

or wedge flow. Now, let us define the generalized Prandtl number for wedge flow as

$$Pr_x = \frac{\rho c_p}{k}\left(\frac{k}{\rho}\right)^{2/(n+1)} (C^3 x_0^{3m-1})^{(n-1)/(n+1)}. \quad (69)$$

The local Nusselt number, $Nu_x$, is then defined by

$$Nu_x = q_w x/k(T_w - T_\infty)$$

$$= C_{mn}(Re_x)^{1/(n+1)}(Pr_x)^{1/3} X^{-1}\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} \quad (70)$$

where

$$C_{mn} = \left\{\frac{a}{6}\frac{[m(2n-1)+1]}{(n+1)}\left[\frac{m(2n-1)+1}{n(n+1)}\right]^{1/(n+1)}\right\}^{1/3},$$

$$X = \left\{1 - \left(\frac{x_0}{x}\right)^{3[m(2n-1)+1]/2(n+1)}\right\}^{1/3},$$

$$b = \left\{\frac{a}{6} n^{2/(n+1)}\left[\frac{n+1}{m(2n-1)+1}\right]^{(1-n)/(n+1)} Pr_x\right\}^{1/3},$$

$$\left(-\frac{\partial\theta}{\partial\zeta}\right)_{\zeta=0} = 1.1198 - \frac{1}{10}\frac{\Lambda}{a^n}\left\{\frac{a}{6} n^{2/(n+1)}\right.$$

$$\left.\times\left[\frac{n+1}{m(2n-1)+1}\right]^{(1-n)/(n+1)}\right\}^{-1/3}$$

$$\times Pr_x^{-1/3} X - [0.018393 + 0.020436(n-1)]\left(\frac{\Lambda}{a^n}\right)^2$$

$$\left\{\frac{a}{6} n^{2/(n+1)}\left[\frac{n+1}{m(2n-1)+1}\right]^{(1-n)/(n+1)}\right\}^{-2/3}$$

$$\times Pr_x^{-2/3} X^2 - \left\{0.12443(n-1)(2\Lambda-1)\right.$$

$$+ [0.0058064 + 0.009677(n-1)]\left(\frac{\Lambda}{a^n}\right)^3$$

$$\times\left[\frac{a}{6} n^{2/(n+1)}\left(\frac{n+1}{2mn-m+1}\right)^{(1-n)/(n+1)}\right]^{-1}$$

$$\times Pr_x^{-1} - 0.0034563\left[(2\Lambda-1)a^{2-n}\right.$$

$$\left. + (n-1)(1-2n)\times\left(\frac{\Lambda}{a^n}\right)^3\right]$$

$$\left[\frac{a}{6} n^{2/(n+1)}\left(\frac{n+1}{2mn-m+1}\right)^{(1-n)/(n+1)}\right]^{-1}$$

$$\left.\times Pr_x^{-1}\right\} x^3 + \ldots$$

Equation (70) reduces exactly to equation (3.5) of Chao [10] for Newtonian wedge flow when $n$ is unity. However, it differs from equation (42a) in Chen *et al.* The difference resulted from the choice of the parameter, $c$ given in equation (34). Chen chose $c = [3(m+1)]/[2(n+1)]$, probably for the reason that the expression of $c$ could reduce to that of Chao *et al.* for the case of Newtonian fluids. The $c$ in Chen *et al.* however, is only good for wedge flow and thus cannot be used for the general geometry since it contains the wedge parameter $m$.

The numerical results for $Nu_x(Re_x)^{-1/(n+1)}$ from the two analyses are tabulated in Table 5 for $n = 0.520$ and 1.60. They agree to within 3% in most cases.

The accuracy is examined again by comparing the present results with those of Lee *et al.* and Acrivos *et al.* The two analyses in the literature have dealt with isothermal surface temperatures, that is to say, $X = 1.0$ in the present analysis. This isothermal temperature distribution is of particular importance in this analysis because the largest error will occur in the series solution of the energy equation at this thermal boundary condition. The agreement of the results between Lee *et al.* and the present method is very good (maximum 0.5% difference) for a right-angle wedge flow as illustrated in Fig. 3. For flow past a flat plate, the discrepancy between Acrivos *et al.* and the present results for the selected values of $n = 0.520$ and 1.60 are 6.9% at maximum. The difference is probably attributed to the number of non-zero terms in the series solution of the present analysis. For this special geometry, the number of non-zero terms in equation (70) reduces to three compared to four non-zero terms

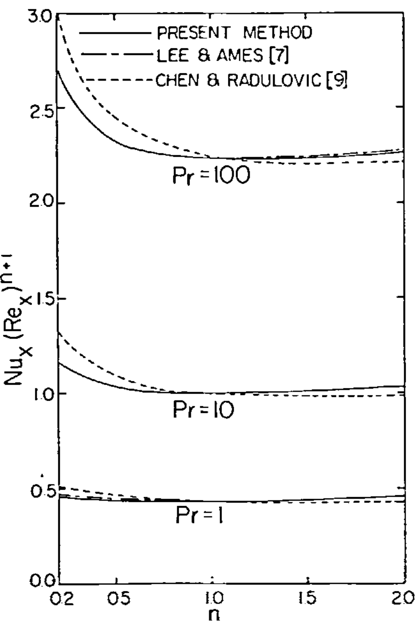


FIG. 3. Comparison of $Nu_x\,(Re_x)^{-1/(n+1)}$ for $\Lambda = 0.5$ and $X = 1.0$.

Table 5. $Nu_x Re_x^{-1/(n+1)}$ for wedge flow $n = 0.520$, $\Lambda = 0.5$

| | $n = 0.520$ | | | | | |
|---|---|---|---|---|---|---|
| | $Pr = 100$ | | $Pr = 10$ | | $Pr = 1$ | |
| $x_0/x$ | present method | Chen [9] | present method | Chen [9] | present method | Chen [9] |
| 0.9 | 5.0721 | 5.0734 | 2.3158 | 2.3168 | 1.0344 | 1.0366 |
| 0.8 | 4.0111 | 4.0154 | 1.8229 | 1.8225 | 1.8046 | 0.8086 |
| 0.7 | 3.4950 | 3.5037 | 1.5830 | 1.5879 | 0.6925 | 0.6984 |
| 0.6 | 3.1689 | 3.1836 | 1.4314 | 1.4392 | 0.6215 | 0.6295 |
| 0.5 | 2.9366 | 2.9593 | 1.3233 | 1.3350 | 0.5708 | 0.5812 |
| 0.4 | 2.7592 | 2.7920 | 1.2408 | 1.2574 | 0.5320 | 0.5453 |
| 0.3 | 2.6174 | 2.6633 | 1.1748 | 1.1976 | 0.5010 | 0.5177 |
| 0.2 | 2.5003 | 2.5631 | 1.1203 | 1.1511 | 0.4753 | 0.4962 |
| 0.1 | 2.4012 | 2.4873 | 1.0742 | 1.1158 | 0.4535 | 0.4800 |
| 0.0 | 2.3158 | 2.4404 | 1.0344 | 1.0940 | 0.4347 | 0.4699 |
| | | | $n = 1.60$ | | | |
| 0.9 | 4.8667 | 4.8662 | 2.2400 | 2.2399 | 1.0189 | 1.0192 |
| 0.8 | 3.8556 | 3.8539 | 1.7703 | 1.7696 | 0.7998 | 0.8000 |
| 0.7 | 3.3638 | 3.3601 | 1.5417 | 1.5402 | 0.6929 | 0.6929 |
| 0.6 | 3.0530 | 3.0466 | 1.3972 | 1.3944 | 0.6252 | 0.6247 |
| 0.5 | 2.8316 | 2.8216 | 1.2942 | 1.2898 | 0.5769 | 0.5757 |
| 0.4 | 2.6625 | 2.6476 | 1.2156 | 1.2088 | 0.5398 | 0.5377 |
| 0.3 | 2.5274 | 2.5057 | 1.1527 | 1.1428 | 0.5102 | 0.5067 |
| 0.2 | 2.4158 | 2.3846 | 1.1007 | 1.0864 | 0.4856 | 0.4801 |
| 0.1 | 2.3214 | 2.2753 | 1.0568 | 1.0355 | 0.4648 | 0.4561 |
| 0.0 | 2.2400 | 2.1537 | 1.0189 | 0.9789 | 0.4468 | 0.4293 |

for other geometries. It is certain that the discrepancy will decrease as more universal functions are carried out in the calculation.

### 5.2. *Flow over a circular cylinder*

Boundary-layer flow over a long horizontal circular cylinder placed in a power-law fluid flow is considered next. For the velocity distribution at the outer edge of boundary layer, the following empirical formula given by Shah *et al* [3] is used in this analysis:

$$\frac{U_e}{U_\infty} = C_1\left(\frac{x}{L}\right) + C_2\left(\frac{x}{L}\right)^3 \tag{71}$$

where $C_1 = 0.92$, $C_2 = -0.131$, and $L$ is the radius of the cylinder. Knowing the velocity distribution outside boundary layer, $\xi$ becomes

$$\xi = \frac{nK}{\rho} U_\infty^{n-2} L^{-n} \int_0^{x/L} (C_1 t + C_2 t^3)^{2n-1}\,\mathrm{d}t. \tag{72}$$

Then, $\Lambda$, $(n+1)\xi\,\mathrm{d}\Lambda/\mathrm{d}\xi$ and $(n+1)^2\xi^2\,\mathrm{d}^2\Lambda/\mathrm{d}\xi^2$ can be subsequently obtained as

$$\Lambda = (n+1)(C_1\delta + C_2\delta^3)^{-2n}(C_1 + 3C_2\delta^2)$$

$$\times \int_0^\delta (C_1 t + C_2 t^3)^{2n-1}\,\mathrm{d}t$$

$$(n+1)\xi\frac{\mathrm{d}\Lambda}{\mathrm{d}\xi} = (n+1)^2\xi\,[(C_1 + 3C_2\delta^2)$$

$$\times (C_1\delta + C_2\delta^3)^{-2n} - 2n(C_1 + 3C_2\delta^2)^2$$

$$\times (C_1\delta + C_2\delta^3)^{-4n}\xi + 6C_2\delta(C_1\delta + C_2\delta^3)^{1-4n}\xi]$$

$$(n+1)^2\xi^2\frac{\mathrm{d}^2\Lambda}{\mathrm{d}\xi^2} = (n+1)^3(\xi)^2[6C_2\delta(C_1\delta + C_2\delta^3)^{1-4n}$$

$$- 4n(C_1 + 3C_2\delta^2)^2(C_1\delta + C_2\delta^3)^{-4n}$$

$$+ 6(1-8n)C_2\delta \times (C_1 + 3C_2\delta^2)(C_1\delta + C_2\delta^3)^{1-6n}\xi$$

$$+ 8n^2(C_1 + 3C_2\delta^2)^3 \times (C_1\delta + C_2\delta^3)^{-6n}\xi$$

$$+ 6C_2(\delta + \xi)(C_1\delta + C_2\delta^3)^{2-6n}]$$

where

$$\delta = \frac{x}{L}.$$

If the local wall friction coefficients, defined in equation (20), are used, then

$$\frac{1}{2}C_f Re^{1/(n+1)} = \left[\frac{1}{n(n+1)\int_0^\delta (C_1 t + C_2 t^3)^{2n-1}\,\mathrm{d}t}\right]^{1/(}$$

$$\times (C_1\delta + C_2\delta^3)^{2n}\left[f_0''(0) + (n+1)\xi\frac{\mathrm{d}\Lambda}{\mathrm{d}\xi}f_1''(0)\right.$$

$$\left. + (n+1)^2\xi^2\frac{\mathrm{d}^2\Lambda}{\mathrm{d}\xi^2}f_2''(0) + \ldots\right]^n. \tag{73}$$

Table 6. $\frac{1}{2} C_f Re^{1/(n+1)}$ for flow over a circular cylinder

| $x/L$ | $n = 0.520$ $\Lambda$ | $\frac{1}{2} C_f Re^{1/(n+1)}$ | $n = 1.60$ $\Lambda$ | $\frac{1}{2} C_f Re^{1/(n+1)}$ |
|---|---|---|---|---|
| 0.1 | 1.4516 | 0.2445 | 0.81591 | 0.05869 |
| 0.2 | 1.4422 | 0.3902 | 0.80826 | 0.1360 |
| 0.4 | 1.3929 | 0.6095 | 0.79029 | 0.3030 |
| 0.6 | 1.3046 | 0.7659 | 0.75854 | 0.4561 |
| 0.8 | 1.1709 | 0.8661 | 0.70657 | 0.5680 |
| 1.0 | 0.9799 | 0.9015 | 0.62321 | 0.6169 |
| 1.2 | 0.7118 | 0.8859 | 0.48668 | 0.5825 |
| 1.4 | 0.3328 | 0.7901 | 0.25137 | 0.4571 |
| 1.5 | 0.0844 | 0.6791 | 0.067955 | 0.3462 |

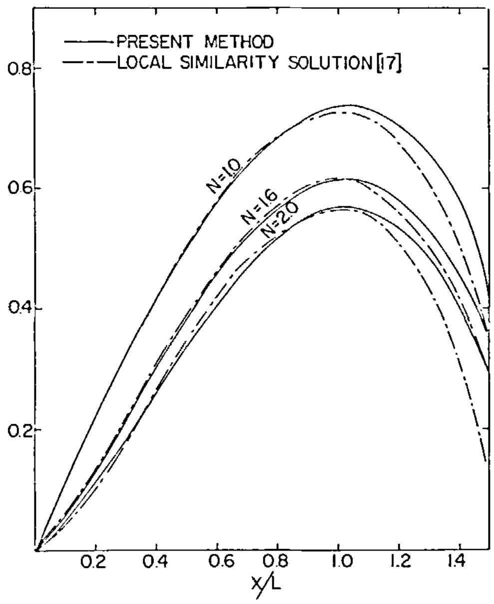


FIG. 4. Comparison of $\frac{1}{2} C_f Re^{1/(n+1)}$ for flow over a circular cylinder.

The numerical values of $1/2\ C_f\ Re^{1/(n+1)}$ are calculated at the specified streamwise location $x$ and tabulated in Table 6.

There are several articles dealing with this type of flow [3, 18]. Recently, Lin *et al.* [17] presented a laminar momentum boundary-layer analysis for power law fluids by using the Merk type of series expansion method. The numerical results of $1/2\ C_f\ Re^{1/(n+1)}$ are compared with those of Lin *et al.* in Fig. 4. They show good agreement in the regions where $x/L$ is small but deviate as the flow develops along the body. This consequence can be predicted because the first term solution, i.e. $f_0$ function, does not contain the perturbation effect even though the two-term velocity distribution of $U_e$ was used in their article.

The rate of heat transfer at the wall, which was expressed in terms of $Nu\ Re^{-1/(n+1)}$ can be obtained by using equation (63). As we have seen in the derivation of the general expression of $Nu\ Re^{-1/(n+1)}$, the expression reduced exactly to that of Jeng *et al.* for Newtonian fluids. Since its accuracy has already been examined and discussed extensively in the case of Newtonian fluids, no attempt will be made to calculate $Nu\ Re^{-1/(n+1)}$ for $n = 1.0$. The two values of $n$ are

Table 7. $Nu\ Re^{-1/(n+1)}$ for flow over a circular cylinder

| $x/L$ | $\Lambda$ | $Pr = 100$ $x_0/L = 0.0$ | $Pr = 100$ $x_0/L = 0.5$ | $Pr = 10$ $x_0/L = 0.0$ | $Pr = 10$ $x_0/L = 0.5$ | $Pr = 1$ $x_0/L = 0.0$ | $Pr = 1$ $x_0/L = 0.5$ |
|---|---|---|---|---|---|---|---|
| | | | | $n = 0.520$ | | | |
| 0.1 | 1.4516 | 2.1373 | — | 0.9548 | — | 0.4043 | — |
| 0.2 | 1.4422 | 2.2763 | — | 1.0106 | — | 0.4203 | — |
| 0.4 | 1.3929 | 2.3919 | — | 1.0547 | — | 0.4308 | — |
| 0.6 | 1.3046 | 2.4137 | 4.3836 | 1.0621 | 1.9779 | 0.4318 | 0.8605 |
| 0.8 | 1.1709 | 2.3731 | 3.2947 | 1.0451 | 1.4737 | 0.4248 | 0.6257 |
| 1.0 | 0.9799 | 2.2651 | 2.8540 | 1.0011 | 1.2750 | 0.4112 | 0.5400 |
| 1.2 | 0.7118 | 2.1218 | 2.5358 | 0.9470 | 1.1394 | 0.3998 | 0.4896 |
| 1.4 | 0.3328 | 1.8976 | 2.1915 | 0.8622 | 1.0291 | 0.3810 | 0.4446 |
| 1.5 | 0.0844 | 1.7027 | 1.9401 | 0.7852 | 0.8954 | 0.3593 | 0.4104 |
| | | | | $n = 1.60$ | | | |
| 0.1 | 0.8112 | 3.5667 | — | 1.6013 | — | 0.6818 | — |
| 0.2 | 0.8071 | 3.3756 | — | 1.5213 | — | 0.6550 | — |
| 0.4 | 0.7900 | 3.1573 | — | 1.4278 | — | 0.6208 | — |
| 0.6 | 0.7584 | 2.9820 | 4.6075 | 1.3512 | 2.1067 | 0.5908 | 0.9440 |
| 0.8 | 0.7065 | 2.8033 | 3.4104 | 1.2724 | 1.5546 | 0.5589 | 0.6908 |
| 1.0 | 0.6232 | 2.6029 | 2.9430 | 1.1936 | 1.3417 | 0.5231 | 0.5965 |
| 1.2 | 0.4867 | 2.3668 | 2.5839 | 1.0794 | 1.1802 | 0.4803 | 0.5273 |
| 1.4 | 0.2514 | 2.0763 | 2.2049 | 0.9531 | 1.0211 | 0.4311 | 0.4627 |
| 1.5 | 0.0680 | 1.8852 | 2.0045 | 0.8717 | 0.9271 | 0.4013 | 0.4270 |

chosen as $n = 0.520$ and 1.60 representing pseudoplastic fluids and dilatant fluids, respectively. The effect of the Prandtl number is also examined by using $Pr = 1$, 10, and 100 for each $n$ value. By using the known information on the wall velocity gradient, $Nu\ Re^{-1/(n+1)}$ can be calculated at any streamwise location for the given value of $X$. As mentioned in the previous section, the upper bound of error will occur at $X = 1$ in the series solution of the temperature field because $X$ varies from zero to one. Thus, the two different surface temperature distributions are chosen as $x_0/L = 0.0$ which corresponds to an isothermal and $x_0/L = 0.5$ for a step change temperature distribution. Since the series is semidivergent at the fourth term, the Euler summation method has been used. The numerical results are tabulated in Table 7.

## 6. CONCLUSIONS

Momentum and heat transfer in power-law fluid flow over arbitrarily shaped 2-dim. or axisymmetrical bodies with non-isothermal surfaces are theoretically examined. The Merk–Meksyn series expansion method and the generalized coordinate transformation can transform the partial differential momentum and energy equations into two sets of infinite-sequence type ordinary differential equations, respectively. The solutions to these sets of differential equations can be obtained as universal functions which are tabulated and for all geometries. The technique presented in this analysis provides a general, accurate, and relatively simple method to analyze the transport phenomena in laminar boundary layer of power-law fluids. In application, the present analysis shows better results than those of ref. [9] even for wedge flow when predicting the rate of heat transfer. Although Lin *et al.* [17] published an analysis of momentum transfer [19], it can be concluded that the present analysis is a significant improvement over the existing literature. Using the results of the present analysis, the heat transfer problem with any arbitrarily-prescribed surface temperature can be analyzed by the method of superposition. The authors also believe that the improved results of the velocity and temperature fields given can significantly improve the prediction of mass transfer in power law fluids with hetrogeneous surface reaction.

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## TRANSFERT DE QUANTITE DE MOUVEMENT ET DE CHALEUR DANS DES FLUIDES A LOI PUISSANCE EN MOUVEMENT AUTOUR DE CORPS BIDIMENSIONNELS OU AXISYMETRIQUES

Résumé—On étudie théoriquement les transferts de quantité de mouvement et de chaleur dans l'écoulement de fluides à loi puissance autour de corps de forme quelconque bidimensionnelle ou axisymétrique. La technique de développement en série du type de Merk est utilisée pour l'analyse de quantité de mouvement. Pour le transfert thermique convectif, une transformation de coordonnées généralisées est employée pour déterminer le champ de température dans la couche limite laminaire pour le corps avec un changement par échelon de la température pariétale. Pour les transferts de quantité de mouvement et de chaleur, la solution des équations est obtenue par des fonctions universelles qui sont indépendantes de la géométrie. Les solutions numériques et analytiques des fonctions universelles sont données et appliquées à l'écoulement autour d'un dièdre et autour d'un cylindre circulaire.

## IMPULS- UND WÄRMEAUSTAUSCH BEI DER STRÖMUNG VON "POWER-LAW" FLUIDEN ÜBER ZWEIDIMENSIONALE ODER ACHSENSYMMETRISCHE KÖRPER

Zusammenfassung—Es wird der Impuls- und Wärmeaustausch bei der Strömung von "power-law"—Fluiden über beliebig gestaltete zwei-dimensionale oder achsensymmetrische Körper theoretisch untersucht. Für die Impulsberechnung wird die Reihenentwicklung nach Merk verwendet. Beim konvektiven Wärmeübergang wird eine verallgemeinerte Koordinatentransformation vorgeschlagen, um das Temperaturfeld in der laminaren Grenzschicht des Körpers bei stufenweiser Änderung der Temperaturverteilung der Oberfläche zu bestimmen. Sowohl beim Impuls- als auch beim Wärmeaustausch erhält man die Lösungen der Erhaltungsgleichungen als universelle Funktionen, die von der Geometrie unabhängig sind. Die Lösungen der universellen Funktionen werden in numerischer und geschlossener Form gewonnen. Sie werden zur Untersuchung der Keilströmung und der Umströmung eines Kreiszylinders angewendet.

## ПЕРЕНОС ИМПУЛЬСА И ТЕПЛА ПРИ ОБТЕКАНИИ ПЛОСКИХ И ОСЕСИММЕТРИЧНЫХ ТЕЛ СТЕПЕННЫМИ ЖИДКОСТЯМИ

Аннотация—Проведено теоретическое исследование переноса импульса и тепла при обтекании степенными жидкостями плоских или осесимметричных тел произвольной формы. Для анализа переноса импульса используется мерковский тип разложения в ряд. В случае конвективного переноса тепла для анализа температурного поля в ламинарном пограничном слое на теле при ступенчатом изменении распределения температуры поверхности используется метод обобщенного преобразования координат. Как для переноса импульса, так и для переноса тепла получено решение основных уравнений в виде универсальных функций, которые не зависят от геометрии задачи. Найдены численное решение и выражение в замкнутой форме для универсальных функций, которые затем используются для анализа обтекания клина и кольцевого цилиндра.